



THE WAKEATRON: ACCELERATION OF ELECTRONS
ON THE WAKE FIELD OF A PROTON BUNCH*

A. G. Ruggiero

October 1984

*Contribution to the Workshop in the New Methods of Acceleration,
Frascati (Rome), Italy, Sept. 25-Oct 1, 1984.

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A.G. Ruggiero

Fermilab, P.O. Box 500, Batavia, IL 60510

Abstract

We expose in this note the idea on how to accelerate a low intensity electron or positron bunch, travelling through a linear rf structure, following at short distance an intense proton bunch which leaves behind a wake field. This device acts like a transformer where two beams are involved: one, made of protons, at high current and low energy, the other, made of either electrons or positrons, at low current and high energy. The two beams are coupled electromagnetically to each other by a specially designed rf structure made of a long sequence of cavities.

We discuss the use of this device for the design of an electron-positron linear collider at 1 TeV energy per beam and luminosity $10^{32} \text{ cm}^{-2} \text{ s}^{-1}$.

Introduction

It has been known for quite some time that a bunched beam in an accelerator, when crossing an rf cavity, leaves behind a wake field along the gap which oscillates at the same frequency (or frequencies) the rf cavity resonates to. This wake field in general persists long enough, especially in the case of cavities with extremely large figures of merit, to affect the motion of subsequent bunches and eventually the next turn around the same bunch that induced it. This phenomenon is called "beam loading". It is well understood and effectively present and visible in all major accelerators or storage ring.

In the case of perfectly conductive rf cavities with no dissipative media, the wake field left behind by a bunch of charged particles is made of energy that will never vanish but will simply oscillate between the inductive and capacitive components (electric and magnetic forms), trapped between the walls of the cavity. A fraction of it will eventually escape by leaking through the opening at the sides of the cavity into the neighboring sections of the vacuum chamber, but most of it will be effectively trapped. There are only two ways the energy can be dissipated so that the wake field will decay at some rate. If the walls are made of resistive material and dissipative media is introduced, currents will be induced in the metal by the wake field and dissipation will occur as consequence of the ohmic relations. Eventually the energy dissipated from its original electro-magnetic form will be all converted in thermal form with a raise of the temperature of the surrounding. This phenomena is also well understood and the figure of merit (Q) of the cavity gives a measure of the size of the effect and therefore of the

decay of the wake field.

It is also possible though that a fraction of the wake field energy, before it has been dissipated considerably, can be recovered by a new bunch of particles following at short distance the one that has generated the wake field. If we call longitudinal the direction along the axis of the rf cavities, and we assume both bunches are travelling at relativistic speed in this direction, a longitudinal electric field must also be generated within the wake field. At the right time interval the phase of this field component will be right to apply an energy impulse to the second bunch, that is to accelerate it.

It has also been known for some time that a bunch of charged particles going across a sequence of rf cavities loses energy by diffraction radiation. An extensive work and research has been done during the sixties and early seventies.^{1,2} A computer code (KN7C) has also been generated to make estimates and analysis was carried out for a variety of cavity geometries. In particular one calculated the energy loss per unit length and per particle and the longitudinal electric field extending behind the bunch of particles. It was found that both these quantities depend linearly with the total charge of the bunch.

At the same time these calculations were done the question arised to many people whether it is possible to accelerate other particles following the exciting bunch at some distance by reabsorbing at least a fraction of the energy lost. The idea has been around for quite sometime until Peryedentsev and Skrinsky wrote a paper³ on the subject and called the device a "Proton Klystron". Among other things, they proposed to accelerate electrons on the wake field of protons, both of them moving in the same direction through a sequence of rf structure. They proposed several configurations on the best use of a beam of protons to load the structure and their idea is still valid and we encourage the reader of this note to read their paper. Nevertheless we believe the reference to a klystron is not really appropriate. Here we give a look on a different approach on how to design a device which accelerate electrons on the wake field of proton bunches. We prefer to call this device a Wakeatron.

General Idea

Before going in too much detail let us first clear a point that must already appear obvious to the reader. Each proton in the leading bunch will lose an amount of energy per unit length that we can write

$$U = \frac{e^2}{\lambda_{Lp}^2} N_p \quad (1)$$

where N_p is the number of protons in the bunch, e the charge either of a proton or an electron, disregarding the sign, and λ_{Lp} an effective length which describes the loss to the rf structure and which depends on the cavities geometry and on the proton bunch dimension and shape. An electron which follows at a convenient distance will gain some energy from that lost by the protons, but will lose also some for the same reason and which will be proportional to the total number N_e of electrons in the bunch. Therefore we can write for the energy gain per unit length

$$W = \frac{e^2}{\lambda_G^2} N_p - \frac{e^2}{\lambda_{Le}^2} N_e \quad (2)$$

Here λ_G is an effective length which describes the energy gain and depends on the geometry of the structure, on the proton bunch dimensions and shape and on the distance of the electrons from the proton bunch. The two lengths λ_{Lp} and λ_{Le} are equivalent and the same, except that the dependence on the proton bunch dimensions and shape is replaced by the dependence on the electron bunch dimensions and shape. It is seen that in order to have an effective (positive) energy gain for the electrons there is a limit on the number N_e of electrons that can be accelerated, that is

$$N_e < \frac{\lambda_{Le}^2}{\lambda_G^2} N_p - \frac{W_0}{e^2/\lambda_{Le}^2} \quad (3)$$

where W_0 is the required energy gain.

In the following we will give some very approximate expressions for λ_G and λ_L . They are very crucial for the design of the Wakeatron and to estimate its performance and efficiency. Clearly a lot of work is in progress and more will be done in the future for their estimate and optimization.

An inspection of both (1) and (2) clearly shows the Wakeatron as a transformer rather than an amplifier (like in the case of a Klystron). Some kind of energy is released to a media and transformed to another kind, the transformation ratio not possibly exceeding unity. Nevertheless the two lengths λ_G and λ_L have different physical meanings and in principle, with the proper configuration of both bunches and with a

proper choice of the cavity geometry, it is possible to accelerate electrons at a rate higher than the rate at which protons lose their energy. Of course not only (3) has to be satisfied but also the conservation of energy: the total power absorbed by the electron bunch cannot be possibly larger than the total power lost by the protons. We can then introduce a transformation efficiency as the ratio of the total energy gained by the electron bunch to the total energy lost by the proton bunch

$$\alpha = \frac{N_e}{N_p} \frac{\lambda_{Lp}^2}{\lambda_G^2} \quad (4)$$

At the very best $\alpha = 1$, which, among other things, assumes perfectly conductive material with no thermal losses. Therefore we like to emphasize again that the careful estimate of λ_G and λ_L , which is not really the scope of this note, is essential.

Outline of the Device

The Wakeatron which we describe next is intended as a linear collider for electron and positrons beams each with energy of 1 TeV and luminosity of $10^{32} \text{cm}^{-2} \text{s}^{-1}$. We believe, as we shall also argue next, that aiming to this performance is reasonable and that it should not really be too difficult to attain. A higher performance is though questionable, but larger energies are certainly possible, the main limitation probably being the amount of real estate one is willing to invest on.

A possible lay-out of the Wakeatron is given in Fig. 1. It is made of two parts identical to each other but arranged symmetrically to each other around the crossing point where the two beams collide. One part is to accelerate electrons and the other positrons. Each part is made of a proton source which generates tight bunches in a conventional way. There is an electron beam source at one side and a positron beam source at the other. The acceleration of electrons and positrons takes place in the two sections of the Wakeatron itself which are identical to each other of the same rf structure. The mode of operation we conceive is that one proton bunch is extracted from each side injected in their respective section of rf structure immediately followed by either an electron or a positron bunch. This will occur at some repetition rate of which all the sources are to be adjusted to.

The Proton Source

Each proton source is made of a Linac, a Booster Ring, an Accelerator Ring and a Shuttle Ring. The parameters for the components are given in Table I. The Accelerator Ring has a cycle of one ramp per second and will deliver 4000 proton bunches to the Shuttle Ring where they will be stored for a period of one second during which they will be extracted one by one until the Shuttle Ring is empty and ready to receive a new load of 4000 bunches from the Accelerator Ring. This mode of operation will provide a constant rate of $f = 4000$ encounters/second. The separation of bunches in the Ring though will be 5 nsec and this obviously is of some concern for the design of an extraction kicker. The proton sources have to deliver for obvious reasons very intense and short bunches. We believe that 10^{11} protons per bunch should be possible with an rms bunch length $\sigma_p = 1$ cm. For comparison the CERN-SPS has obtained 1.3×10^{11} protons in a bunch with $\sigma_p < 10$ cm. The short bunch length can be achieved by raising the transition energy and choosing large accelerating rf frequency and voltage. The proton beam energy has been set to 100 GeV according to the following criterion. As we shall see a luminosity figure of $10^{32} \text{ cm}^{-2} \text{ s}^{-1}$ will require a power of about 1 MW in either electron or positron beam with a repetition rate of $f = 4000 \text{ s}^{-1}$; for efficiency consideration the proton beam power ought to be much larger than this and we have chosen 10 MW which yields the energy we have selected for the protons as shown also in Table I. From this point of view, the minimum proton beam energy is 10 GeV, unless one is willing to increase the number of particles per bunch well beyond what we believe is practical. On the other hand it is obvious that larger energies are even more desirable because not only more power would be made available, but also the bunch dimensions can be furtherly reduced. We believe that the energy of 100 GeV represents a good compromise when one also considers cost, magnets and required repetition rate. The parameters list given in Table I is intended as an example. The design of the whole proton source waits clearly a more detailed study which will make some of these parameters to change in order to meet the beam specifications. In particular we believe that an individual bunch area of 0.5 eV-sec is large enough for the beam to be stable against microwave coherent excitation, assuming a coupling impedance $|Z/n| \sim 1$ ohm. Also the rms emittance, the same in both planes, $\epsilon = \sigma^2/\beta_L = (10^{-6})/\gamma$ is consistent with the assumptions for other projects of the moment (SSC, LHC).

The Source for Electrons and Positrons

The electrons and positrons sources are each made of a linac and a damping ring. To generate positrons a target is inserted between the two devices. Parameters are given in Table II, and are intended as just an example. A more careful analysis is required to optimize the design of the two sources. Anyway, in principal, the linac will generate bunches at the rate $f = 4000$ per second. To produce positrons efficiently an energy of 1.0 GeV should be more than adequate and the linac could be made operating in the S-band mode like SLAC which would give an overall length of about 100 m at a gradient of 10 MV/m.

The electron/positron bunches are then transferred in the damping ring where they are kept circulating for a while. The main function of the damping ring is to hold the beam until it is "cooled" effectively by synchrotron radiation. We propose that there are 100 bunches at any time circulating in the ring; since bunches are to be extracted at the rate of $f = 4000$ per second, the time each bunch will spend in the ring is $100/f = 25$ msec. For the radiation effects to take over effectively we require the typical radiation damping time corresponds to a fraction of the circulating time, for example $1/3$ which is 8 msec. The parameters given in Table III would yield roughly an equilibrium rms emittance $\epsilon = \sigma^2/\beta_L = (10^{-6}\text{m})/\gamma$ similar to the proton beam. For the electron/positron bunch length we have taken an rms value $\sigma_e = 1\text{mm}$ smaller than the length of the proton bunches. The design of these sources should be rather straight-forward and conventional.

The Wakeatron

Fig. 2 shows the geometry of the rf structure that makes the two sections of the Wakeatron. It is made of a sequence of a very large number of identical cells with cylindrical geometry. Both beams go through a central circular opening of diameter $2a$. The outer radius is b and assumed to be much larger than the gap width g . The walls of the cavities are taken to be perfectly conductive with no thermal losses and their thickness negligible compared to the gap width. The interior of the cavities is filled with good vacuum.

A bunch of N particles each with electric charge e , rms longitudinal length σ_L and practically no transverse dimensions loses an amount of energy when traversing the rf energy, that is given by eq. (1) when expressed per unit length and per particle. It has

been estimated by several people¹ that the effective length is

$$\lambda_L = \sqrt{2} a \exp(\sigma_1^2/2g^2) \quad (5)$$

where we have also introduced an exponential factor to take into account the longitudinal extension of the bunch. Similarly it is also possible to estimate the amplitude of the wake field.² It is speculated that the energy gain for a particle following the primary bunch has a form given by (2), with the effective length λ_G given by

$$\lambda_G = \frac{a}{\epsilon} \exp(\sigma_1^2/4g^2) \quad (6)$$

In both (5) and (6) σ_1 is the rms length of the bunch inducing the wake field. They are expected to be valid in the limit $b \rightarrow \infty$, that is for the case each cavity is made of two infinite parallel planes. In eq. (6), ϵ is a form factor which depends on the cavity geometry, the distribution of the primary bunch and on the distance between this and a following particle to be accelerated. It is expected $\epsilon < 1$, but this factor still requires a better analysis and it is certainly one of the major parameters for investigation. In particular it is possible that ϵ depends somewhat on the gap width g and on the opening radius a , also in the limit $b \rightarrow \infty$. For the time being we will consider this constant as such and independent of all other parameters; a point that waits crucially for verification.

Inspection of both (5) and (6) shows already some results:

- (i) Since one requires λ_G and λ_L small, it is seen that a small opening is more effective. In principle this requires the smallest beam dimensions. The emittance for both beams are given in Tables I and II and vertical as well as horizontal focussing has to be provided along the Wakeatron, eventually with external means. Obviously the proton beam has the largest transverse dimension; one can achieve an rms cross-section $\sigma = 0.3$ mm if the maximum value of the amplitude lattice function $\beta_{\max} = 10$ m, which is not impossible. The electron/positron bunch transverse dimensions are expected to be smaller. Of course extreme care must be taken so that all the beams involved are kept stable against coherent excitation from beam loading and transverse modes, and that their eventual emittance growth can be effectively controlled to the desired value. To

allow enough room for the proton beam we propose here $a = 1$ mm. It is possible for instance to make the rf cavities as a stack of a large number of conductive, parallel plates and let the proton beam itself to drill a hole through the structure.

- (ii) A very weak dependence on the energy is expected for both λ_L and λ_G as long as the two beams are travelling at relativistic velocities.
- (iii) It seems that the gap width g enters only in the exponential factor where it is compared to the rms bunch length σ_1 . One requires $\sigma_1 < g$ in which case both the energy loss and the energy gain per unit length and per particle do not seem to depend on the gap width g , at least as long $b \gg g$. This is a point that requires verification and intensive study and it could be connected to the definition of the form factor ϵ .
- (iv) It may seem that the exponential factors introduced in eqs. (5) and (6) are ad hoc. Actually they can be explicitly derived.^{1,2} Observe the different dependence on the exponent in the two equations; the dependence on σ_1 is weaker for the energy gain. This can be easily explained by recalling that the energy loss is an integral over the cavity volume of the square of the electric field, whereas the energy gain is the electric field itself.

With (5) and (6), the transformation coefficient (4) becomes

$$\alpha = 2\epsilon^2 \frac{N_e}{N_p} \exp(\sigma_1^2/2g^2) \quad (7)$$

As we have said at the most $\alpha = 1$; in reality a fraction of the energy left behind by the proton bunch will be dissipated in other ways. We assume here $\alpha = 0.4$; an assumption that ought to be confirmed. Since, as we shall see later $N_e = 3 \times 10^9$ particles per bunch are required for a luminosity of $10^{32} \text{ cm}^{-2} \text{ s}^{-1}$ at 1 TeV we need

$$\epsilon^2 \exp(\sigma_1^2/2g^2) = 6.7 \quad (8)$$

The rf cavities ought to be designed with a gap g so that (8) is fulfilled. This remains to be proven. For the moment we can guess and propose

$$g = 1 \text{ cm}$$

$$\varepsilon = 2.0$$

and then it is proper to take for instance $b = 20 \text{ cm}$. The thickness of the material can be taken to be 1 mm or less. All the other parameters can be easily derived and they are listed in Table III. In particular the energy gain expected is 350 MeV/m which corresponds to a length of almost 3 Km to achieve an energy 1 TeV . The protons would lose about 26.5 MeV/m , that is a final energy of about 25 GeV at the end of the Wakeatron.

A problem at this point arises. It is necessary that the two beams have as equal velocities as possible so that the distance between the two bunches does not change more than a fraction of the electron bunch length over the length of the accelerating structure. If this is 3 Km , synchronism between the two beams is completely lost. A single argument is the following: in the case $\gamma_e \gg \gamma_p$ and both are constant, the relative difference for the lengths travelled is

$$\frac{\Delta L}{L} \sim \frac{1}{2\gamma_p^2}$$

Even if we take $\gamma_p = 100$ and $L = 3000 \text{ m}$ we obtain $\Delta L = 15 \text{ cm}$, which is too large. To cope with this problem the scheme outlined in Fig. 1 should be changed; a possibility being to divide the Wakeatron into several short stages next to each other. Each stage should have a length short enough to preserve synchronism between the two beams. Synchronism is then restored from one stage to another with either multiple bunch operation,³ or by adjusting the path length of the two beams.

Another problem also of very serious concern is to keep both beams moving right on the axis of the rf structure. Do not so one avoids excitation of transverse modes and the possible instabilities that these can cause. It is certainly crucial to preserve the normalized emittance generated by the electron/positron damping rings. We assume that this is possible.

The performance of the linear collider is described in Table IV. With the present scheme where the energy of the protons is 100 GeV it is not possible to raise the energy of the electrons or positrons beyond 1 TeV . To do this larger proton energy is required. For instance to generate a $10 \text{ TeV } e^\pm$ beam it seems that a 1 TeV proton beam is required.

To preserve the repetition rate of 4000 pulses per second the proton source dimensions have also to increase correspondingly ($R \sim 10$ Km). If one can do this then also the luminosity will increase if one assumes that the chosen normalized emittance is a truly invariant. For instance a linear collider of 10 TeV x 10 TeV could generate a luminosity of $10^{33} \text{cm}^{-2} \text{s}^{-1}$.

Efficiency and Cost Considerations

We expect the following requirements for the power needed to operate the collider.

Each Proton Source:	
Beam Power	10 MW
Magnet Power	10 MW
RF Power	10 MW
TOTAL	30 MW

Each Electron Source:	
Linac	3 MW
Damping Ring	3 MW
TOTAL	6 MW

Few MW will be also required for focussing and transport along the Wakeatron. Therefore we estimate a total of $2 \times 40 = 80$ MW to operate the entire device with the exclusion of the detector. The electron or positron beam we have described above is equivalent to a power of 2 MW (for each beam) and the efficiency can be estimated as the ratio $(2 \times 2 \text{ MW}) / 80 \text{ W} = 1/20$, which is not a bad figure at all.

One can make also a very rough estimate for the cost of the entire device (again, excluding the detector, in the following way):

Each Proton Source	200 M\$
Each Electron Source	25 M\$
Each Section of RF Structure	100 M\$
Transport, Transfer and Focussing for Each Side	25 M\$
TOTAL	350 M\$
	x 2
TOTAL PROJECT	700 M\$

free of any contingency and escalation.

References

1. E. Keil, Proc. Intern. Conf. High Energy Accelerators, II (Yerewan, 1969), p. 551. Also Nuclear Instr. and Methods 100 (1972), p. 419.

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3. E.A. Peryedenstev and A.N. Skrinsky. Proc. of the 6th All-Union Conference on Charged Particles Accelerators (Dubna, 1978), Dubna, 1979, v. 2, p. 272. Also: Proceed. of the 12th Int. Conf. on High-Energy Accelerator. Fermilab, Aug. 11-16, 1983, p. 508.

Table I Parameters for the Proton Source

Components:	Energy	Radius
Linac	1 GeV	-
Booster	1-10 GeV	100 m
Accelerator Ring	10-100 GeV	1000 m
Shuttle Ring	100 GeV	1000 m

Cycle Rate:

Booster	30 Hz
Accelerator Ring	1 Hz

RF	Frequency	Voltage
Booster	175-200 MHz	2 MV
Accelerator Ring	200 MHz	7 MV
Shuttle Ring	200 MHz	10 MV

Beam Parameters in the Shuttle Ring:

No. of bunches (= rf harmonic no.)	4000
Bunch-to-bunch separation	5 nsec
Rms bunch length	1 cm
Long phase space area (95% of beam)	0.5 eV-sec
No. of protons/bunch	10^{11}
Normalized emittance (H and V) σ^2/β_L (rms)	10^{-6} m

Table II Electron Positron Beam Sources

Linac:	
Output Energy	1 GeV
RF	3 GHz
Damping Ring:	
Energy	1 GeV
Average Radius	10 m
Packing Factor	50%
Dipole Field	8 kg
Betatron tune, $\nu_{H,V}$	~15
no. of bunches circulating	100
No. of particles/bunch	3×10^9
Radiation damping time	8 msec
σ^2/β_L (H and V, assuming full coupling)	10^{-6} m/ γ_e
RMS bunch length	1 mm
Time interval between individual bunch extraction	25 msec
Energy Loss	20 KeV/turn
RF:	
Frequency	500 MHz
Voltage	100 kV

Table III The Wakeatron RF Structure

Cavities:	
Gap width, g	1 cm
Iris radius, a	1 mm
Outer radius, b	20 cm
Wall thickness (copper)	<1 mm
Effective loss length, λ_{Lp}	2.33 m
Effective gain length, λ_{Gp}	0.642 mm
Transformation coefficient, α	0.4
Form factor, ϵ	2.0
Energy Loss per proton	26.5 MeV/m
Energy Gain per electron/positron	350 MeV/m
Energy Loss per electron/positron	1.3 MeV/m
Total length of the rf-structure	2x3 km

Table IV Collider Performance

Luminosity	$10^{32} \text{ cm}^{-2} \text{ s}^{-1}$
Repetition rate, f	4 KHz
No. of particles/bunch, N	3×10^9
$\beta_{H,V}^*$	5 mm
RMS β -emittance, σ^2/β	
H and V, assuming full coupling	$10^{-6} \text{ m}/\gamma_e$
Energy per beam	1 TeV
Rms beam spot, $\sigma_{H,V}^*$	500 \AA
Rms bunch length, σ_e	1 mm
Disruption parameter, D	1
Energy spread due to Beamstrahlung	3%
Power in each beam	2 MW

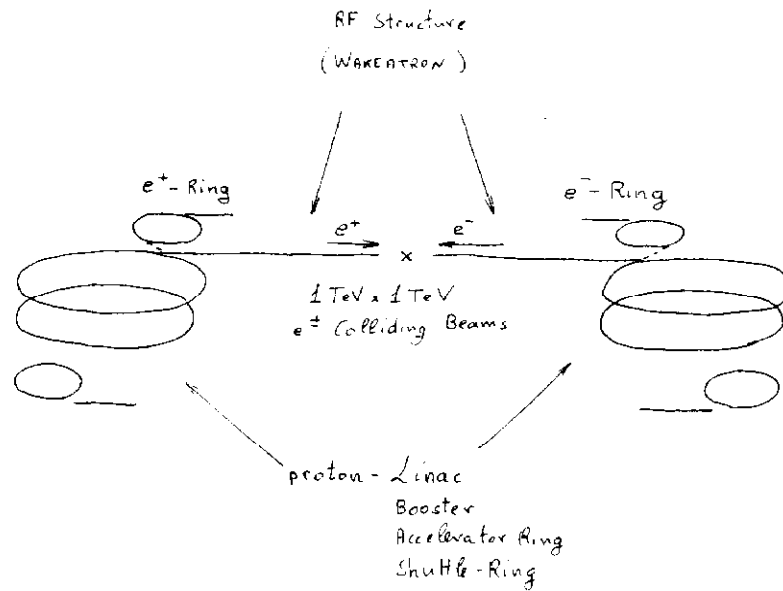


FIG. 1 LAY-OUT OF AN ELECTRON-POSITRON LINEAR COLLIDER WHICH MAKES USE OF THE IDEA OF ACCELERATING ELECTRONS AND POSITRONS ON THE WAKE FIELD OF INTENSE PROTON BUNCHES.

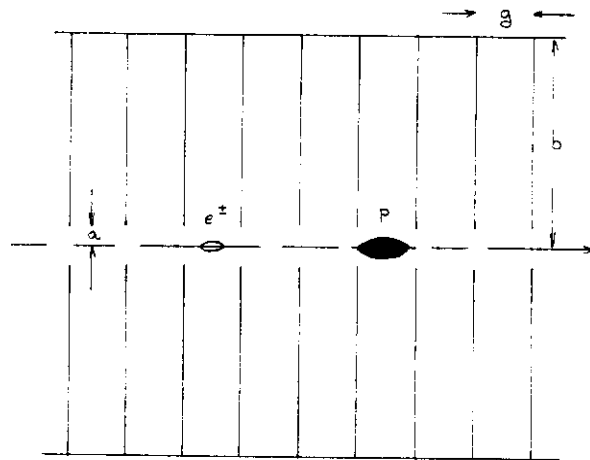


FIG. 2 CONCEPTUAL DESIGN OF AN RF STRUCTURE TO ACCELERATE ELECTRONS OR POSITRONS ON THE WAKE FIELD OF AN INTENSE PROTON BUNCH (PROTON KLYSTRON, WAKEATRON).